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DEVELOPMENT OF MODELS AND ALGORITHMS FOR STUDYING THE DYNAMICS OF MULTIDIMENSIONAL SYSTEMS WITH PULSE-WIDTH MODULATION

The problems of synthesizing optimal control actions in systems with pulse-width modulation encounters difficulties associated with the obtained sufficiently simple expressions, which gives a correct solution, intended use of the results in control problems for complex discrete dynamic objects in real time. The use of existing methods does not give acceptable results even with the use of modern computing facilities, due to the cumbersomeness of the results obtained, as well as the difficulty of ensuring the physical realizability of the expressions obtained. In this regard, the development of machine-oriented methods for the synthesis of optimal control actions in a system with pulse-width modulation, the use of which does not require a large mathematical calculation, and having a large degree of formalization, is undoubtedly an urgent scientific and technical problem.

Keywords — nonlinear signal modulation, interpretation of the dynamics of impulse systems, iterative search for control actions, pulse-width modulation, optimization problem, microprocessor controller.

Quite a lot of works, methods and algorithms are devoted to the solution of the problem of synthesis of optimal control actions in systems with linear modulation of signals [1]. The main disadvantages of these methods are their extremely cumbersome and complex mathematical apparatus, a large number of simplifying sentences and calculations, and the complexity of interpreting the results obtained. In addition, the use of these methods often leads to obtaining systems of partial differential equations or algebraic transcendental equations, the exact solution of which is impossible: if some calculations are incorrect, in principle, there may be no solution; the use of numerical methods for solving with a large dimension of the resulting system, even when using the capabilities of modern computers, can give an absolutely unacceptable result [2].

As an example, below is a simulation model of a pulse width modulator (PWM). The considered PWM circuit consists of pulse width modulators and a continuous linear part. The duration of the n-th pulse at the output of each of the modulators $i = 1,2,\dots N$ is determined by the value of the error signal $e(nT_i)$ calculated at discrete moments of time, i.e.

$$\tau_n^i = \begin{cases} \varphi^i[e(nT_i)] \text{ при } \varphi^i[e(nT_i)] \le T_i, \\ T \text{ при } \varphi^i[e(nT_i)] > T_i, \end{cases}$$

where T_i - pulse repetition period at the PWM output; φ^i - modulation characteristic of the width modulator.

The use of this method for systems with nonlinear modulation of the control action requires a modification of the known algorithm for solving the problem of transferring a multidimensional linear dynamic plant with M input and N output controlled variables from a given initial state to the required final state in a minimum number of control cycles. It is assumed that the sampling period is the same for all input signals. The minimum possible number of translation ticks in accordance with the N-interval theorem is determined by the expression:

$$L = Int\left\{\sum_{i=1}^{N}\sum_{j=1}^{M} P_{ij} / M + 0.5\right\} \tag{1}$$

where P_{ij} - order of the transfer function (differential equation) of the channel; j -th entrance; i -th output of the control object.

The required state of the control object is determined by the conditions

$$Y_i(L+K) = G_i(L+K), i = \overline{1,N}; K = \overline{0,\tilde{N}_i}, \tag{2}$$

where $Y_i(L+K)$ - is the value of the i -th output variable in the $(L+K)$ -th cycle; $G_i(L+K)$ - the required value of the i -th output variable; G_i -the number of cycles of fixing the i -th output variable.

Based on the analysis of the dynamics of the behavior of the control object, we will change conditions (2) to the following::

$$Y_i(L+K) = E_i(L+K), i = \overline{1,N}; K = \overline{0,\tilde{N}_i}, \tag{3}$$

where

$$E_i(L+K) = G_i(L+K) - Y_i^*(L+K), \tag{4}$$

$Y_i^*(L+K)$ - the predicted value of the i -th output variable provided:

$$U_j(m) = 0, j = \overline{1,M}; m = \overline{1,L} \tag{5}$$

In the case of zero initial conditions, these dependencies will have the form:

$$Y_i(L+K) = \sum_{j=1}^{M}\sum_{m=1}^{L} U_j(m) * \omega_{ij}((L+K-m+1)*T), \quad i = \overline{1,N}; K = \overline{0,C_i}, \tag{6}$$

where T - control signal sampling period; $\omega(qT)$ - the value of the weighting function (response to a pulse of duration T) in the q -th cycle.

Combining the system of expressions (6) with conditions (3), we obtain the system of linear algebraic equations:

$$W * U = E \tag{7}$$

where W - weighting function coefficient matrix:

$$W = \lfloor \omega_{ij}(L + K - m + 1) \rfloor; \tag{8}$$

U - vector column of predicted error values:

$$U = \begin{bmatrix} U_1(1), U_2(2), ..., U_1(L), U_2(1), ... \\ U_2(L), U_m(1), ..., U_m(L) \end{bmatrix}^T \tag{9}$$

E column vector of predicted error values:

$$A = \begin{bmatrix} A_1(1), A_1(2), ..., A_1(L), \\ A_2(1), ..., A_2(\tilde{N}_2 + L), A_n(L), ..., E_n(C_i + L) \end{bmatrix}^T \tag{10}$$

The dimension of system (7) is:

$$M * L = \sum_{i=1}^{N} C_i \tag{11}$$

Having solved system (7), we obtain the desired control actions in the form of linear combinations of predicted errors:

$$U_j(m) = \sum_{i=1}^{N} \sum_{K=0}^{C_1} R_{im} (\sum_{s=1}^{i=1} C_s + K) * E_i(L + K) \tag{12}$$

where R_{im} - vector row matrix ω^{-1}.

The obtained expressions (12) are actually the main correcting procedure in the iterative search for control actions modulated in width. But first, let us consider some necessary conditions, the fulfillment of which should ensure the solution of the task. First, the pulse repetition period for pulse width modulation must be equal to the sampling period of the control signal during synthesis for a linear pulse system. Secondly, the condition must be met:

$$|U_j(m)| < A_j \tag{13}$$

where A_j - amplitude of width modulated control actions.

Let us assume that the solution of the synthesis problem for a linear impulsive system with the number of translation cycles determined by expression (1) led to the failure of condition (13). Let's increase the number of translation ticks by J. Let's take it first $J = 1$. Then the values of the predicted errors change and will be determined by the expression:

$$E_i^j(L + K) = E_i(L + K) - \sum_{j=1}^{M} \sum_{m=1}^{J} U_j(m) * \omega_{ij}(L + K - m + 1), \tag{14}$$

$$i = \overline{1, N}; K = \overline{1; C_i + J}$$

We substitute the found expressions for the predicted errors for expression (12), which allows us to express L of the main control actions for each input variable J through additional controls

$$U_j(J+m) = U_j(j+m) + \sum_{K=1}^{M}\sum_{i=1}^{J}\omega_{ij}(L+k-m+1+J)*U_k(i) \quad (15)$$

Now it is necessary to solve the optimization problem associated with minimizing the criterion:

$$F = \sum_{j=1}^{M}\sum_{i=1}^{L+J}U_j^2(i) \to \min; (j=\overline{1,M}; i=\overline{1,J}) \quad (16)$$

This problem is solved simply by using the least squares method. As a result of its solution, the values of auxiliary control actions are found $U_j(k),(k=1,J)$. .If they all satisfy condition (13), then using formula (14) we find the values of the predicted errors, substitute them into expression (12) and find the values of control actions $U_j(k),(k=\overline{J+1,L+J})$.They also need to be checked for the fulfillment of condition (13). If it is fulfilled, then you can proceed to the next stage of the synthesis. Otherwise, the value of J must be increased by one and the procedure for minimizing the sum of the squares of the control actions must be repeated.

In the article, the method is based on the representation of the dynamics of impulse systems in the form of a space of state variables and the use of the N-interval theorem, which allows the system to be transferred to the required state in a minimum number of control cycles. The use of nonlinear modulation of signals of the pulse-width modulation type, based on an increase in control cycles, taking into account the predicted control errors during an iterative search for control actions modulated along the width. The solution to this problem is based on the proposition that the total area of control pulses for each output for a linear pulse system and a system with pulse-width control modulation must be equal. In this case, the values of the control actions found during the synthesis for a linear impulse system are corrections in the form of a change in the area of the corresponding control signals, modulated in width. The found control actions provide a control error of less than 3%.

References

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